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The Time Evolution of Beam in the Recycler Ring

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Abstract

In this report, we study the time evolution of beam in the Recycler Ring due to vacuum gases, noise and other generic processes. We obtain expressions for the beam profile, lifetime and emittance growth based on initial beam conditions. We develop a fitting procedure to obtain the diffusion coefficient and lifetime due to processes causing abrupt loss of beam particles.

Basic Formulation

For most lifetime measurements, we introduce a thin beam current in the middle of RR aperture. In such cases, the beam lifetime and evolution are determined by two classes of processes: (1) Processes like single coloumb scattering, nuclear scattering or ionization by which the beam loses a particle abruptly at any given time and situation; (2) Diffusion processes like multiple coloumb scattering, intrabeam scattering or some form of noise where the beam emittance grows and eventually the beam loses particles by hitting the aperture. For the time being, we assume that these processes are independent and write the beam current at any given time as:

$$I(t) = I_0 N_{ab}(t) N_{df}(t)$$

where I_0 is the initial beam introduced, $N_{ab}(t)$ denotes time evolution due to abrupt loss of beam particles as in the first case, and $N_{df}(t)$ denotes the time evolution due to diffusion prcesses as described in the second case.

For most cases, we can write:

$$N_{ab}(t) = e^{-\frac{t}{\tau_{ab}}}$$

where τ_{ab} characterize the lifetime due to processes belong to the first case. Now to describe the diffusive processes, we have to solve the Foker-Planck equation :

$$\frac{\partial f}{\partial \tau} = \frac{\partial}{\partial Z}(Z\frac{\partial f}{\partial Z})$$

where f describe the particle ditribution and subject to the boundary conditions:

$$f(Z,0) = f_0(Z)$$
$$f(1,\tau) = 0$$

where Z $= \epsilon/\epsilon_a =$ emittance/acceptance, and $\tau =$ tR$/\epsilon_a$ with R, the diffusion coefficient. For just multiple coloumb scattering phenomena, the diffusion coefficient R is given in terms of scattering angle θ by:

$$R = \beta_{avg}\langle\dot{\theta}^2\rangle$$

The general solution of the above equation can be written as:

$$f(Z,\tau) = \sum_n C_n J_0(\lambda_n\sqrt{Z})e^{-\lambda_n^2\tau/4}$$

with coefficients C_n:

$$C_n = \frac{1}{J_1(\lambda_n)^2}\int_0^1 f_0(Z)J_0(\lambda_n\sqrt{Z})dZ$$

where λ_n is nth root of the Bessel function $J_0(Z)$. Now we can obtain the total beam particles as a function of time:

$$N_{df}(t) = \int_0^1 f(Z,\tau)dZ = 2\sum_n \frac{C_n}{\lambda_n}J_1(\lambda_n)e^{-(\lambda_n^2 R/4\epsilon_a)t}$$

The beam lifetime can be computed using:

$$\tau_{mc} = -\frac{N(\tau)}{dN(\tau)/d\tau}$$

The beam life time varies with time and normally reaches an asymptotic value:

$$\tau_a = \frac{4\epsilon_a}{\lambda_1^2 R}$$

.

The emittance growth can be obtained from:

$$\frac{d\epsilon_N}{dt} = \frac{\pi\beta\gamma}{2} R$$

Now combining the expressions for N_{ab} and N_{df}, we can write the evolution of beam current as:

$$I(t) = I_0 e^{-\frac{t}{\tau_{ab}}} 2 \sum_n \frac{C_n}{\lambda_n} J_1(\lambda_n) e^{-(\lambda_n^2 R/4\epsilon_a)t}$$

Since the beam current measurement as function of time is most accurate measurement we can make in the Recycler Ring, we can fit the measurements for two parameters - for the diffusion constant R and the lifetime due to abrupt processes τ_{ab}. From these, further information about the vacuum or other processes can be extracted. This method also eliminates the reliance on emittance growth measurements that are believed to problematic! Or simply, we can use this method as a cross check or another handle on understanding of the relevant physical quantities.

Time Evolution and Beam Profile - Examples

We simulate the time evolution of a beam introduced in the Recycler Ring with a gaussian shape in the phase space given by:

$$f(Z) = \frac{a^2}{2\sigma^2} e^{-}(a^2/2\sigma^2)Z$$

with $Z = \epsilon/\epsilon_a$ ranging from 0.0 to 1.0. Here ϵ, ϵ_a denotes emittance and Recyler Ring acceptance respectively and a is the half aperture equal to the average radius of the beam pipe (0.023 m). The basic parameters used for the simulation are given in Table 1. We also vary these parameters for further understanding of the time evolution and associated lifetime issues.

Parameter	Value
RR Acceptance	40.0 π-mm-mr
Initial beam σ	0.004 m
Lifetime due to abrupt processes	500000 s ($\sim$ 139 hours)
Diffusion constant	3.00E-10
Average lattice β	40.0 m
Average beam $\beta\gamma$	9.46
RR Half Aperture	0.023 m

Table 1: The Recycler Ring parameters used for baseline simulation

Following figures illustrate the beam shape, beam current and lifetime with various values of basic parameters listed in the above table.

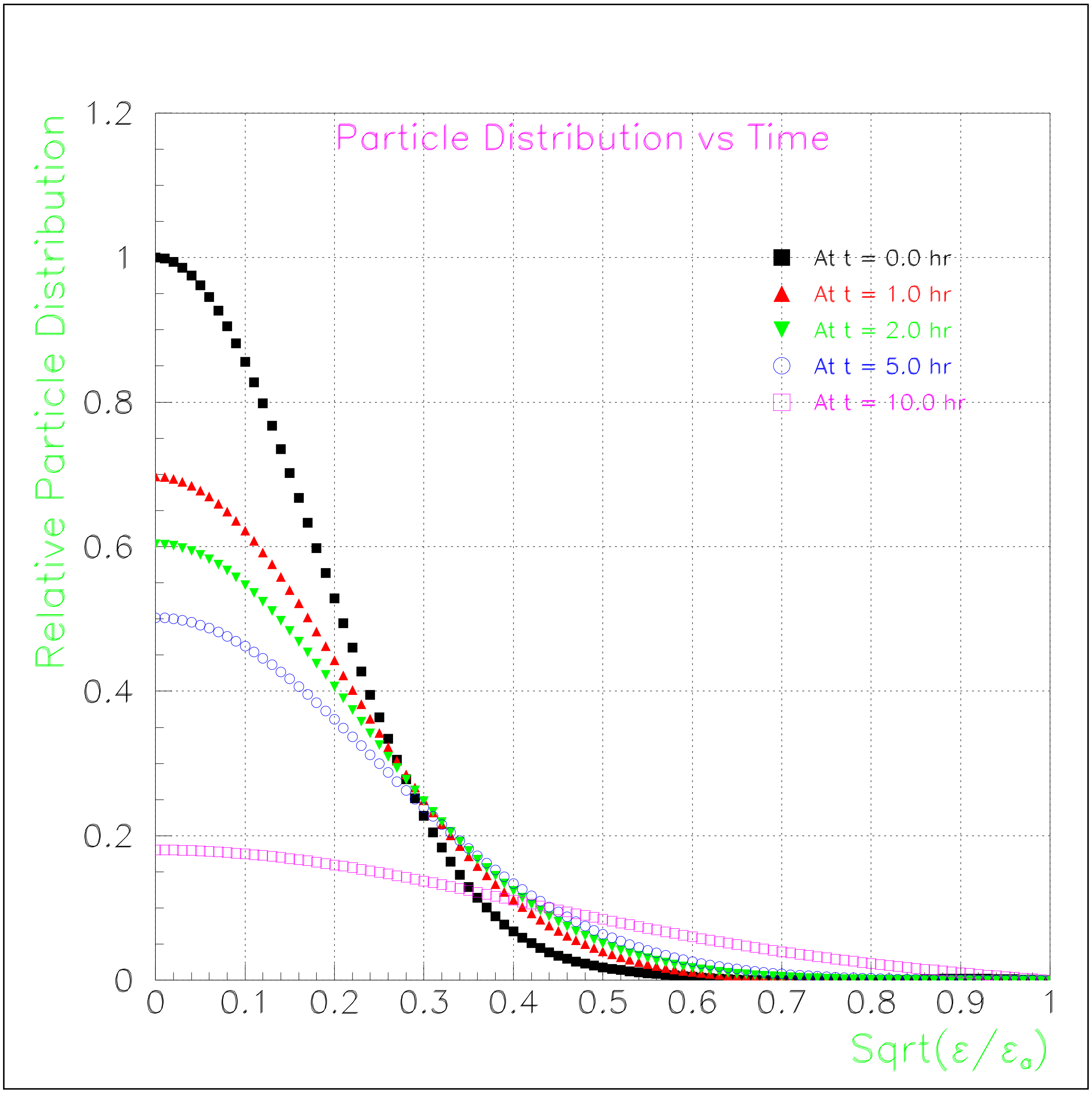

Figure 1: Beam profile at various times: at initial stage, after 1, 2, 5, and 10 hours. The parameters used for this simulation are listed in Table 1.

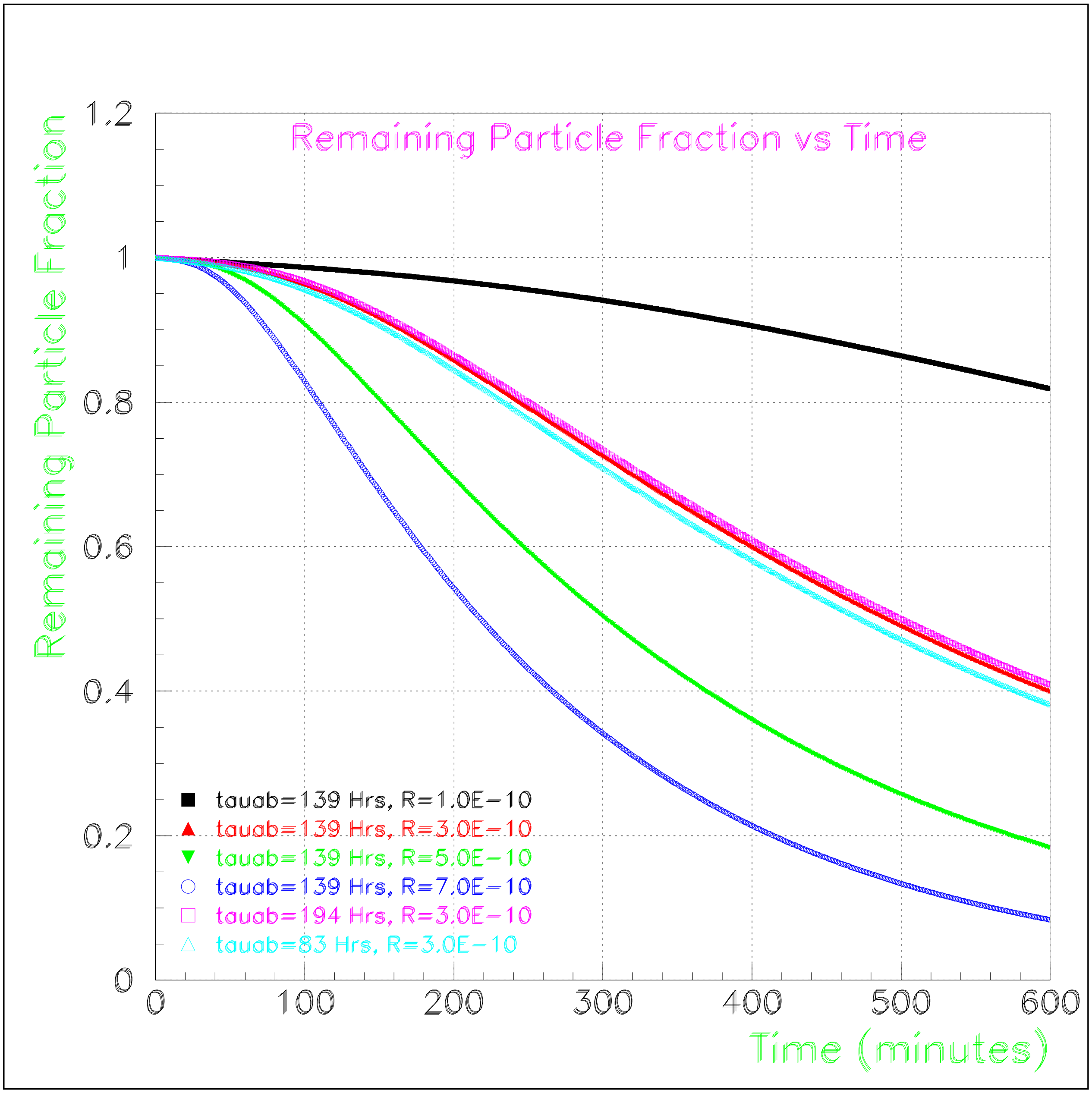

Figure 2: The beam current time evolution for various cases of diffusion constant R and τ_{ab} - the lifetime due to abrupt processes.

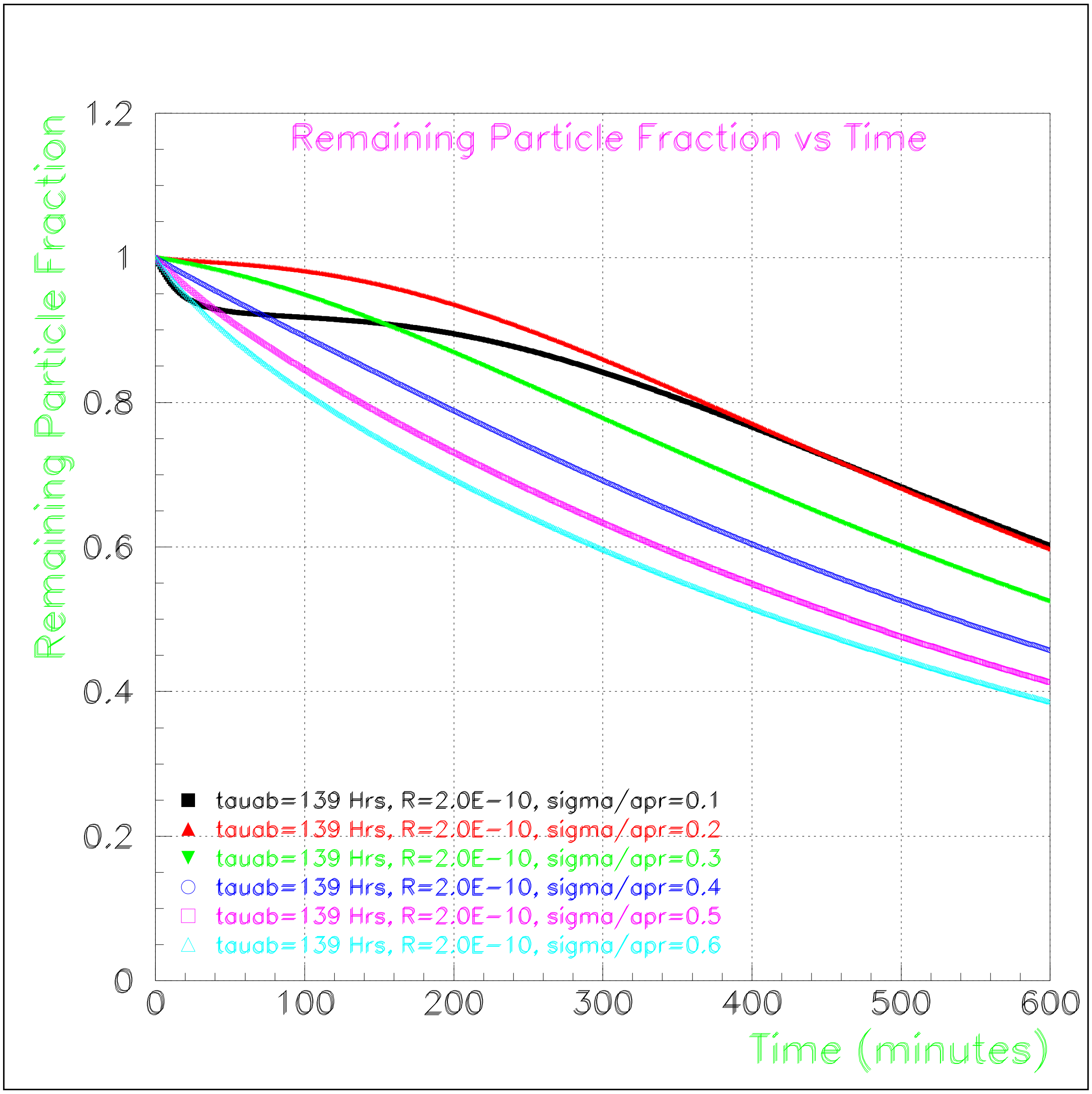

Figure 3: The beam current time evolution as a function of initial beam σ - for σ/(Half aperture) = 0.1 to 0.6

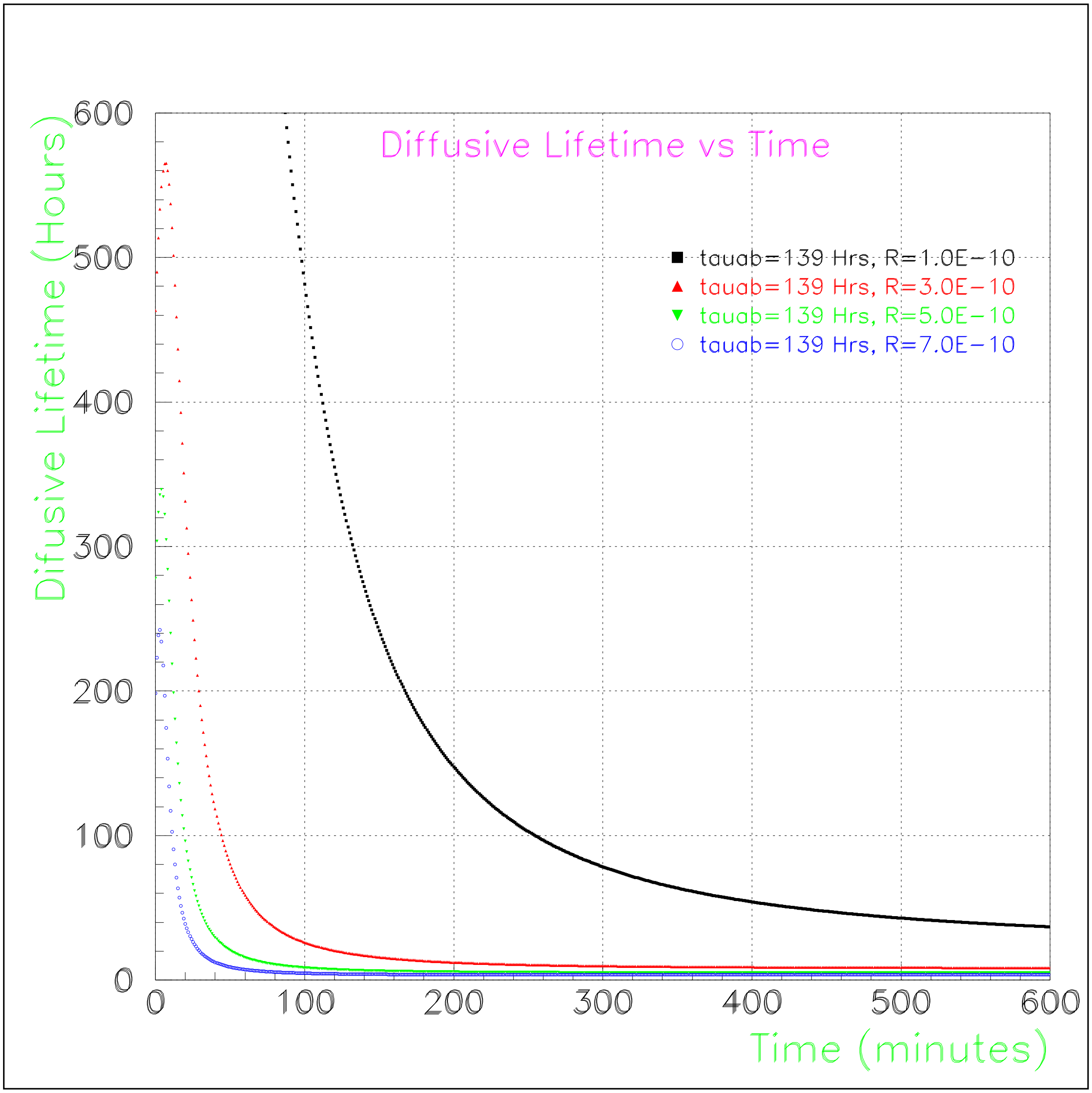

Figure 4: The time evolution of beam lifetime due to diffusive processes for various values of diffusion constant R

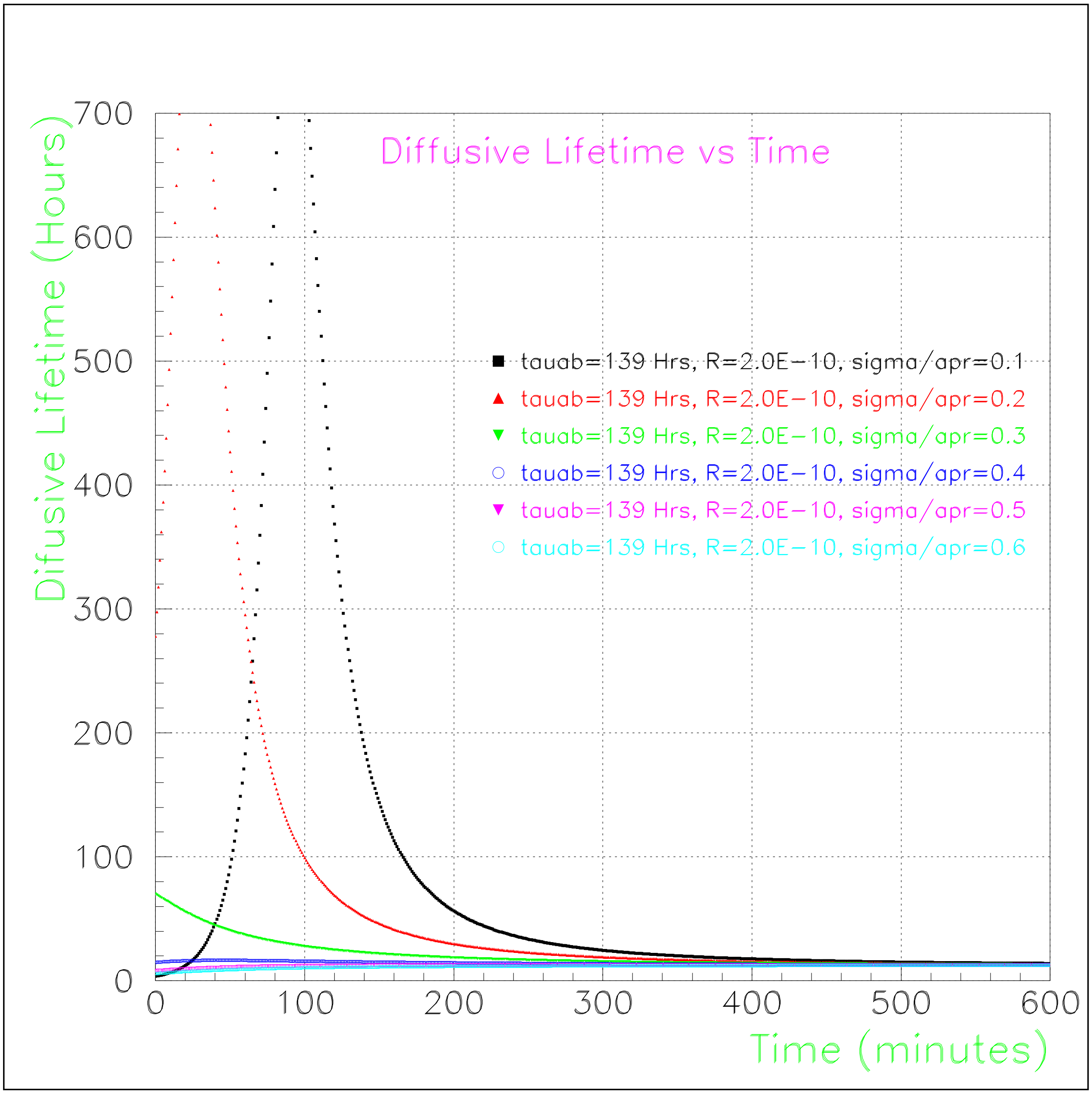

Figure 5: The time evolution of beam lifetime due to diffusive processes for various values of σ/(Half aperture)

Fitting Procedure

To apply the above formalism to real Recycler Ring data, we need to develop a fitting procedure. From the above formalism, the beam current at a given time t after t = 0 can be written as:

$$I(t) = I_0 e^{-\frac{t}{\tau_{ab}}} \sum_n Y_n e^{-R\alpha_n t}$$

with the coefficients $Y_n = Y_n(\frac{\sigma}{a}, \epsilon_a)$, and $\alpha_n = \alpha_n(\epsilon_a)$. The coefficients Y_n, α_n can be generated numerically for most cases once knowing the initial beam σ, the RR acceptance ϵ_a and half aperture a. We can obtain I(t) - same as 'IBEAM' measured. We can fit for I(t) for τ_{ab} and R using any normal fitting technique. Once knowing R, we can extrapolate to many quantities such as computing the emittance growth using:

$$\frac{d\epsilon_N}{dt} = \frac{\pi\beta\gamma}{2} R$$

To illustrate the technique, we simulated I(t) as in the previous section and fitted with a PAW (CERN/HBOOK analysis software) based least square technique. The values used to generate the distribution and the resultant fitted values are compared in the following table.

For most practical puposes, the sum in the above expression for I(t) can be limited to first 5 terms or less. This can be easily seen from zeros of the Bessel function $J_0(X)$ in increasing order: 2.405, 5.520, 8.654, 11.792, and 14.931. Here we use first five terms of the above expansion.

Parameters Used for Generation	Parameters Obtained By Fitting
σ=0.005 m, τ_{ab}=5.0E+5 s, R=2.0E-10	τ_{ab}=(4.88$\pm$0.12)E+5 s, R=(2.21$\pm$0.20)E-10
σ=0.005 m, τ_{ab}=5.0E+5 s, R=4.0E-10	τ_{ab}=(4.88$\pm$0.19)E+5 s, R=(4.00$\pm$0.05)E-10
σ=0.005 m, τ_{ab}=8.0E+5 s, R=3.0E-10	τ_{ab}=(7.41$\pm$0.88)E+5 s, R=(3.01$\pm$0.05)E-10
σ=0.005 m, τ_{ab}=5.0E+5 s, R=3.0E-10	τ_{ab}=(4.88$\pm$0.21)E+5 s, R=(3.19$\pm$0.26)E-10
σ=0.007 m, τ_{ab}=5.0E+5 s, R=3.0E-10	τ_{ab}=(4.88$\pm$0.14)E+5 s, R=(3.18$\pm$0.27)E-10
σ=0.009 m, τ_{ab}=5.0E+5 s, R=3.0E-10	τ_{ab}=(4.88$\pm$0.24)E+5 s, R=(3.18$\pm$0.28)E-10

Table 2: Comparison of paramater vaules used for generation and those obatined from fitting

Some comments about fitting I(t) for R and τ_{ab} might be relevant here. Here we are trying to obtain quantities of the order of E+5 (τ_{ab} in seconds), and E-10 (diffusion constant in MKS units). Therefore proper numerical scaling as well as specification of probable parameter space have to be included in the fitting program for convergence.

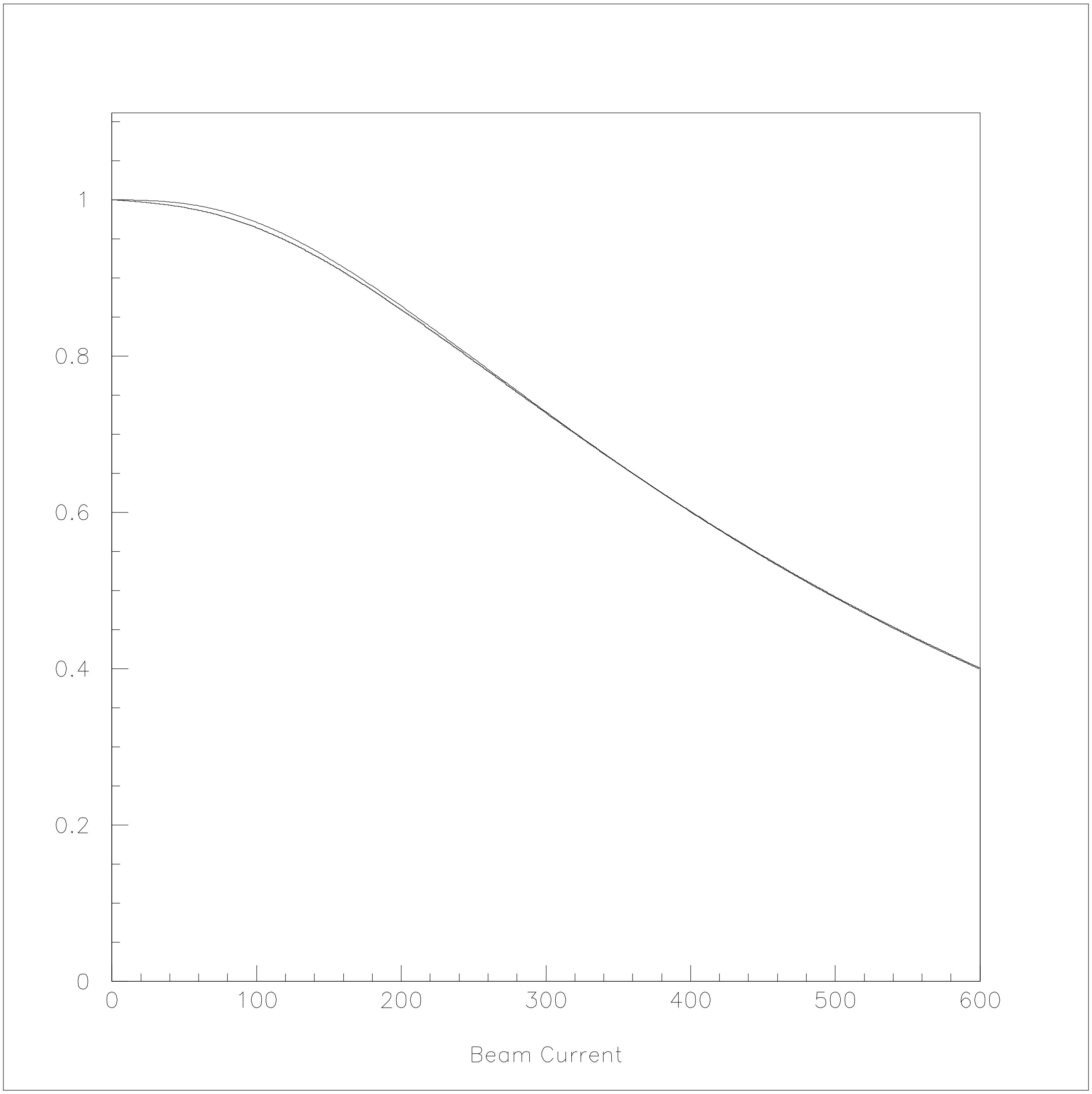

Figure 6: The beam current time evolution for τ_{ab}=5.0E+5 s, R = 3.0E-10 along with fitted curve. The values obtained by fitting are: $\tau_{ab} = (4.88 \pm 0.21)$E+5 s; R = (3.19 ± 0.26)E-10.

Recycler Ring Beam - Results

Still to be completed after real measurements.

Summary and Outlook

Still to be completed.